Input Impedance of Rectangular Microstrip Patch Antenna With Iso/Anisopropic Substrate-Superstrate

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***Abstract*—The modified Wolff model (MWM), which is an improved version of the cavity model, is presented to compute the resonance frequency and input impedance of a rectangular microstrip patch antenna under the isotropic/anisotropic substrate-superstrate configuration. The model has accuracy better than 1.5% for both the resonance frequency and resonant resistance, as compared against the results of the method of moments (MOM) and experimental results.**

***Index Terms*—Anisotropic patch antenna, microstrip antenna, multilayer patch antenna.**

I. Introduction

THE original cavity model for the microstrip patch antenna as proposed by Lo *et al*[1] has limited accuracy. In its original form, it does not handle a microstrip patch under the isotropic and anisotropic multilayer dielectric medium. Using the concept of dynamic relative permittivity introduced by Wolff and Knoppik [2], Verma and Rostamy [3] developed the modified Wolff model (MWM) to take care of the patch antenna under the multilayer dielectric medium.

The present communication, with help of the MWM, extends the original cavity model to compute the input impedance of a probe-fed rectangular microstrip patch antenna with isotropic/anisotropic substrate-superstrate shown in Fig. 1. The results of a full-wave analysis and experimental results are not available for the input impedance of a patch on an anisotropic dielectric substrate. Therefore, results for the input impedance computed by the present model are compared against the results of the method of moments (MOM) for the isotropic case only. However, the MWM computes resonance frequency of a patch on the anisotropic substrate with accuracy within 1.5%, as compared against both the experimental results and the results of the full-wave analysis.

II. Resonance Frequency

Fig. 1 shows a shielded rectangular microstrip patch in an uniaxial anisotropic substrate-superstrate configuration. The relative permittivity tensor of the uniaxial anisotropic dielectric is given by

$$\bar{\varepsilon}_{ri} = \begin{bmatrix} \varepsilon_{xxi} & \varepsilon_{xyi} \\ \varepsilon_{yxi} & \varepsilon_{yyi} \end{bmatrix} \quad (1)$$

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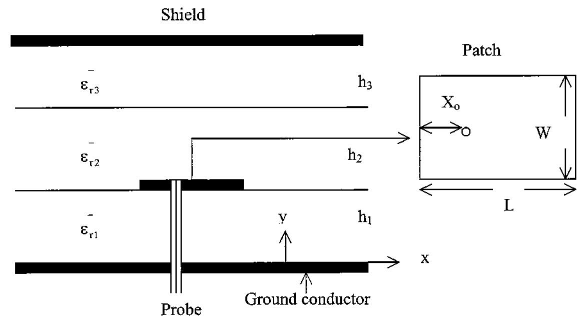

Fig. 1. Probe-fed rectangular patch antenna on anisotropic superstrate.

where, $i = 1, 2, 3$ indicates the dielectric layers. The anisotropic substrate and superstrate layers are replaced by the equivalent isotropic dielectric layers with equivalent relative permittivity $\varepsilon_{r,\text{eq}i}$ and equivalent height $h_{\text{eq}i}$ given by [4]

$$\varepsilon_{r\text{eq}i} = \sqrt{\varepsilon_{xxi}\varepsilon_{yyi} - \varepsilon_{xyi}^2} \quad (2)$$

$$h_{\text{eq}i} = h_i\sqrt{\frac{\varepsilon_{xxi}}{\varepsilon_{yyi}} - \left(\frac{\varepsilon_{xyi}}{\varepsilon_{yyi}}\right)^2} \quad (3)$$

with $i = 1$ for the substrate and $i = 2, 3$ for the superstrate layers. When optic axis of the uniaxial crystal is aligned perpendicular to the plane of the patch, $\varepsilon_{xyi} = 0$.

The MWM can be used to determine the resonance frequency f_r of a patch in the equivalent isotropic dielectric medium [3]

$$f_r = \frac{v_0}{2\sqrt{\varepsilon_{r\,\text{dyn}}}}\left[\left(\frac{n}{L_{\text{eff}}}\right)^2 + \left(\frac{m}{W_{\text{eff}}}\right)^2\right]^{\frac{1}{2}} \quad (4)$$

where v_0 is the velocity of an electromagnetic wave in the free-space, and n and m are the modal numbers along the length (L) and width (W) of the patch respectively. Determination of the dynamic effective relative permittivity $(\varepsilon_{r\,\text{dyn}})$, the effective length (L_{eff}), and the effective width (W_{eff}) has been discussed in [3], where $\varepsilon_{r\,\text{eq}i}$ and $h_{\text{eq}i}(i = 1, 2, 3)$ in place of ε_{ri} and h_i have been used to take care of anisoptropy in the dielectric layers.

III. Input Impedance

The microstrip patch can be modeled as a parallel resonant circuit. The frequency dependent input impedance of the probe fed microstrip patch with superstrate is given by

$$Z_{\text{in}}(f) = \frac{R_{\text{res}}}{1 + Q_T^2\left[\frac{f}{f_r} - \frac{f_r}{f}\right]} + j\left[X_L - \frac{R_{\text{res}}Q_T\left[\frac{f}{f_r} - \frac{f_r}{f}\right]}{1 + Q_T^2\left[\frac{f}{f_r} - \frac{f_r}{f}\right]^2}\right] \quad (5)$$

where f is the frequency.

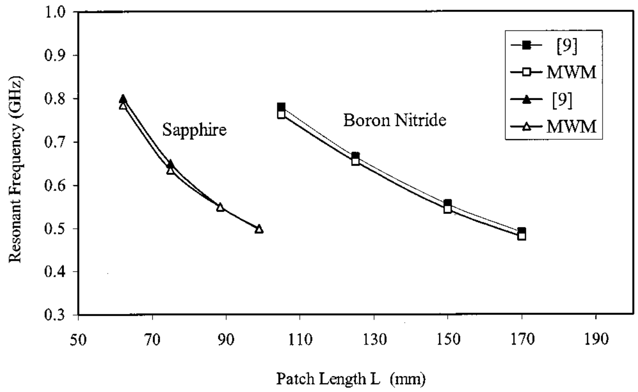

Fig. 2. Resonant frequency of microstrip resonator on anisotropic substrates with h1=12.7 mm, h3=88.9mm, h2=0mm, and W=20.0 mm

The input resonant resistance $R_{\rm res}$ at the feed point X_o along length of the patch as shown in Fig. 1 and the inductive reactance of probe, X_L indicating effect of all nonresonating modes are given by

$$R_{\rm res} = \frac{Q_T h_{\rm eq1}}{\pi f_r \varepsilon_{r\,\rm dyn} \varepsilon_0 LW} \cos^2\left(\frac{\pi}{L} X_o\right) \quad (6)$$

$$X_L = \frac{377 f_r h_{\rm eq1}}{v_0} \log\left[\frac{v_0}{\pi f_r\, d_o \sqrt{\varepsilon_{r\,\rm dyn}}}\right] \quad (7)$$

where d_o is the diameter of co-axial feed and $h_{\rm eq1}$ is the equivalent substrate thickness between the patch and ground plane and f_r is the resonance frequency of the patch.

The total Q-factor Q_T of the cavity is obtained from

$$\frac{1}{Q_T} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_R} \quad (8)$$

where Q_c, Q_d and Q_R are the Q-factors corresponding to the conductor loss, dielectric loss, and radiation loss from the cavity walls [5] respectively. These Q-factors are given by

$$Q_c = \frac{\pi f \sqrt{\varepsilon'_{r\,\rm eff}}}{v_0 \alpha_c}, \quad Q_d = \frac{\pi f \sqrt{\varepsilon'_{r\,\rm eff}}}{v_0 \alpha_d}, \quad Q_R = \frac{v_0 \sqrt{\varepsilon_{r\,\rm dyn}}}{4 f_r h_{\rm eq1}}. \quad (9)$$

The loss coefficients α_c and α_d corresponding to the conductor and dielectric losses for the structure shown in Fig. 1 are obtained by the SLR-formulation [6], [7]. The $\varepsilon'_{r\,\rm eff}$ is the real part of complex $\varepsilon^*_{r\,\rm eff}$ obtained by the variational method. The $\varepsilon'_{r\,\rm eff}$ is taken as average of $\varepsilon'_{r\,\rm eff}$ obtained from both the width and length sides. The computed value of $\varepsilon'_{r\,\rm eff}$ has been improved by adding a correction factor K, i.e., $\varepsilon'_{r\,\rm eff}(1+K)$. The computation of $\varepsilon'_{r\,\rm eff}$ with correction factor is discussed in the Ref. [8].

IV. Results and Discussion

The structure shown in Fig. 1 can be reduced to an open microstrip patch on the anisotropic substrate for $h_2 = 0.0, \bar{\varepsilon}_{r2} = 1, \bar{\varepsilon}_{r3} = 1$ and by taking the shield far away i.e., $h_3 \gg h_1$. Fig. 2 compares the resonance frequency of a rectangular patch on the sapphire ($\varepsilon_{xx1} = 11.6, \varepsilon_{yy1} = 9.4$) and boron nitride ($\varepsilon_{xx1} = 3.4, \varepsilon_{yy1} = 5.12$) substrates computed by the MWM and by the SDA [9]. The MWM results follow the results of SDA within 1.5%. Table I further compares accuracy of the MWM against the experiment results and also against the results of MOM of Pozar, *et al.* [10] for the patch on epsilam-10 ($\varepsilon_{xx1} = 13.0, \varepsilon_{yy1} = 10.2$) substrate. Accuracy of the MWM is comparable to that of the accuracy of MOM and it is also within 1.5% of the experimental results.

TABLE I
Resonant Frequency of Rectangular Microstrip Antennas on Epsilam-10 Substrates ($\varepsilon_{xx1} = 13.0, \varepsilon_{yy1} = 10.2$)

(ε_{xx1} =13.0, ε_{yy1} =10.2).

h_1 (mm)	L (mm)	W (mm)	Expt. [10] GHz	Pozar [10] GHz	MWM GHz	% error MWM	% error Pozar
1.27	20.0	30.0	2.264	2.268	2.23	1.5	0.2
1.27	9.5	15.0	4.495	4.52	4.472	0.5	0.6
2.54	19.0	30.0	2.242	2.26	2.236	0.27	0.8

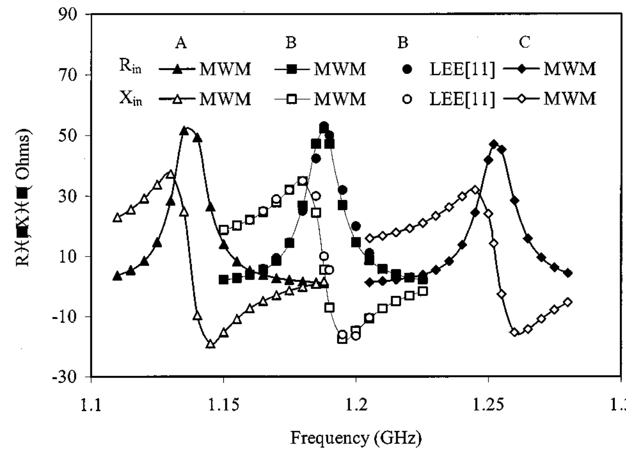

Fig. 3. Input impedance of rectangular patch on anisotropic substrate.

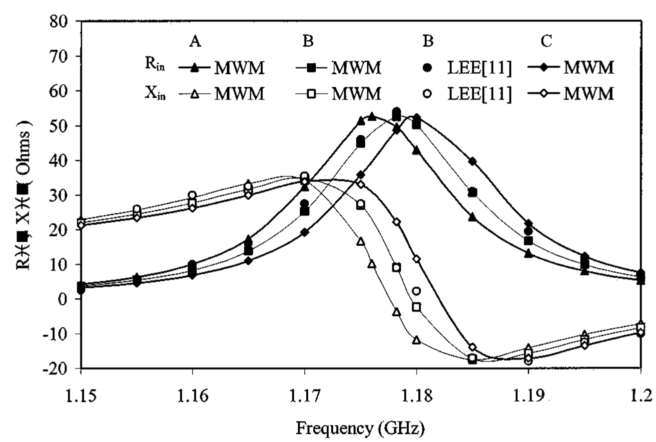

Fig. 4. Input impedance of rectangular patch on isotropic substrate and anisotropic superstrate.

Fig. 3 shows the input impedance of a rectangular patch on an anisotropic substrate with anisotropic ratio $\varepsilon_{xx1}/\varepsilon_{yy1} = 0.8, 1.0, 1.2$ corresponding to the positive uniaxial, isotropic, and negative uniaxial substrates. For three cases, the relative permittivity components are given as case A: $\varepsilon_{xx1} = 3.168, \varepsilon_{yy1} = 2.64$; case B: $\varepsilon_{xx1} = \varepsilon_{yy1} = 2.64$; case C: $\varepsilon_{xx1} = 2.112, \varepsilon_{yy1} = 2.64$, respectively. Other design parameters are $W = 114.3$ mm, $L = 76.2$ mm, $h_1 = 1.59$ mm, feed location $X_o = 22.9$ mm, inner diameter of probe $d_o = 1.27$ mm, and $\tan\delta_1 = 0.003$. For the isotropic case,

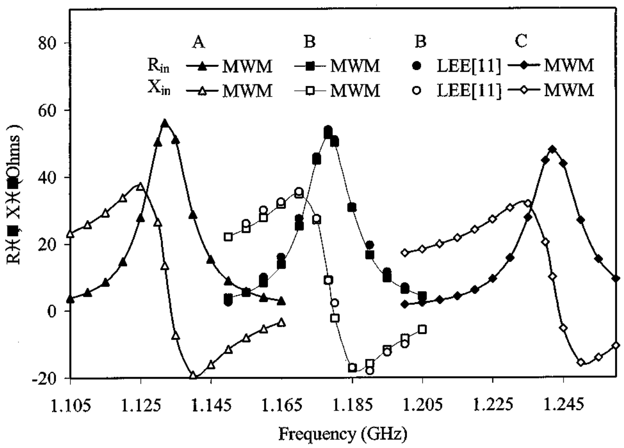

Fig. 5. Input impedance of rectangular patch on anisotropic substrate and isotropic superstrate.

the results for R_{in} and X_{in} obtained by the MWM and results of the MOM, obtained by Lee, *et al.* [11], are almost identical with only 0.02% deviation in the resonance frequency and 1.4 % deviation in the R_{in} at resonance.

Fig. 1 can also be reduced to a patch antenna with superstrate by taking $h_3 = \propto$ and $\bar{\varepsilon}_{r3} = 1$. Figs. 4 and 5 show the results on R_{in} and X_{in} for the patch on isotropic substrate-anisotropic superstrate and the patch on anisotropic substrate-isotropic superstrate, respectively. When both the substrate and superstrate are isotropic dielectrics, the MWM results are compared against the result of MOM of Lee, *et al.* [11] giving deviation 0.04% and 2.6% in the resonance frequency and in the R_{in} at resonance respectively. From Fig. 5, we find that anisotropy in the substrate has more pronounced effect on the resonance frequency and also on the resonant resistance as compared to anisotropy in the superstrate shown in Fig. 4. Thus anisotropy in substrate can take the narrow bandwidth patch antenna out of the operating band, whereas its effect on the resonant resistance may not degrade the return loss very much.

V. Conclusion

We have presented an improved cavity called MWM, which is capable of determining both the resonance frequency and the input impedance for the rectangular patch antenna under the iso/anisotropic substrate-superstrate configuration. The model has accuracy of the full wave methods and is computationally much faster than the numerical methods. The MWM is based upon the variational formulation in the spectral domain, which takes into account the multilayer dielectric medium in its Green's function [3]. Therefore, the present form of MWM can also accommodate more number of dielectric layers in the cavity model.

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